Cosmic Chatter

Thoughts on space and human exploration

A SHORT HISTORY OF PI

March 14th - written 3/14 here in the US - is Pi Day! One of the most remarkable numbers in mathematics (a field chock full of amazing ones!), pi was the first special number discovered and plays a fundamental role in physics today. I have a special fascination with it, and I'd like to share a little of that with you. So, hang on, this is going to be pretty cool.

First off, a definition: pi is the ratio of the distance around a circle's edge (its *circumference*) and the distance across it (its *diameter*). If you draw a circle, no matter how big or small, it will have this exact same ratio: about 3.14159265358979323846264338327950288841971...

This fact has been known since antiquity. Written records from ancient Babylon and Egypt show that we knew the value of pi to within one percent nearly 4,000 years ago. The Babylonians knew it as 3.125, the Egyptians as 3.1605—not bad! How were these early values found? Probably the same way you or I would think to do it: by drawing a circle and measuring the two dimensions. Approximations were common: 25/8, 256/81, and 22/7 are all pretty close.

The first recorded mathematical approach to finding the true value of pi was developed by the famous Greek mathematician Archimedes around 250 BCE. He realized that a circle is really just a multi-sided object with an infinite number of sides. By starting with a triangle and doubling the number of sides, he got a hexagon. Doubling again gave him a dodecagon. He continued doubling the sides until he had 96. Even though that's a lot of sides, he could measure their total length precisely and come up with an estimate of pi. He proved that pi was larger than 3.1408 and smaller than 3.1429. That's just as accurate as the value (3.14) that we still teach students today!

The ancient Greeks' fascination with pi led them to pose one of the most enigmatic mathematical puzzles of all time. Could one, using nothing but a compass and an unmarked straightedge, construct a square with exactly the same area as a given circle? Mathematicians would try for more than two thousand years to accomplish this task, to no avail.

Archimedes' geometric approach to computing pi would be the standard for 1600 years. The next breakthrough would come in India. Around 1400 CE an Indian mathematician named Madhava of Sangamagrama used a new technique: the infinite series. Infinite series are one of the more curious phenomena in mathematics: never-ending strings of numbers which, when added together, yield a finite result. Madhava developed one such series and used it to compute pi to 11 decimal places. For all practical purposes, the exploration of pi was complete. Not until the development of the atomic bomb and the space race would more digits be necessary.

Although the practical development of pi was now complete, the truly fascinating properties of this number wouldn't be discovered for centuries. In 1781, Swiss scientist Johann Lambert proved that pi was irrational. This means that the digits of pi never end and that there is no systematic repetition. Remember squaring the circle and the two thousand years of struggle to accomplish it? No one could do it because it can't be done: no technique can yield the straight lines of a square with exactly the length needed to satisfy a number that never ends!

The development of pi would touch nearly every great mathematician in history. It's no surprise, then, that history's greatest mathematician would use pi in its most remarkable application. In 1748, Leonhard Euler developed a formula relating the exponential function to the well-known functions sine and cosine. A special case of this formula, known today as Euler's Identity, occurs when you set the value to pi. Euler showed that:

$$e^{i\pi} + 1 = 0$$

This formula contains the five most important numbers in mathematics: *e*, *i*, *pi*, 0, and 1. *e*, Euler's number, is another irrational, never-ending number like pi. *i*, the imaginary number, is the (otherwise impossible) square root of -1. Zero is the *additive constant*, adding it to any number gives you the same number back. Finally, one is the *multiplicative constant*; multiplying any number by it also gives you the same number back. Here we are, combining two never-ending numbers and one imaginary one, adding 1 and we get back 0! Incredible.

By the dawn of the 20th century, computation of pi had begun to stall out. The constant was now known to more than 500 decimal places - a feat which took 15 years of computation! It would take the development of computers to advance things once more. In the aftermath of World War II, the first multi-purpose computer, ENIAC, was used to compute pi to more than 3,000 decimal places in just three days. Like everything else involving computers, that figure has grown astonishingly quickly. Today, we know pi to more than twelve billion digits.

In a way, our understanding of pi has traced the development of civilization. From the early use of measuring tools, to the development of geometry and the discovery of calculus, to our modern use of computers, we've applied our most powerful tools to the understanding of a number which we now know underlies many aspects of our existence. So, take a moment today and think, what has pi done for me?

March 14, 2014 / Morgan Rehnberg
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Mohankumar 4 years ago · 0 Likes

A Polygon with however small finite chain of straight line segments shall never be a circle and hence Archimedes' value of Pi remains an approximation with however much number of digits. The ancient Greek's hence posed the most mathematical puzzles of using nothing a compass and an unmarked straightedge to construct a square with exactly the same area as a given circle. This may be achieved only with Babylonian value of Pi, which is also a rational value. (√2 x 1.25) = √3.125 & (√3.125 / 2)^2 = 0.78125. Unfortunately, Greek's werent aware of Babylonian value of pi at that point of time.